Technical Comments

Comment on "Aerodynamic Estimation Techniques for Aerostats and Airships"

B. Etkin* and G. D'Eleuterio†
Institute for Aerospace Studies
University of Toronto, Ontario, Canada

JONES and DeLaurier[1] have undertaken a difficult task in developing methods for estimating the aerodynamic forces on aerostats and airships. We trust that their results will be of practical utility to workers in this field. However, the portion of their paper that deals with the modeling of forces in unsteady flow has errors in the theory that users should be aware of. In particular, Eq. (45) of the paper is wrong. It reads

$$\dot{w} = \dot{W} + \dot{P}y - \dot{Q}x - Pv + Qu \tag{1}$$

In fact, w is defined correctly in Eq. (14) of Ref. 1, and its time rate of change $\dot{w}$ (for a quasirigid vehicle with $\dot{y} = \dot{x} \approx 0$) is

$$\dot{w} = \dot{W} + \dot{P}y - \dot{Q}x \tag{2}$$

This follows clearly and simply from the definitions of the quantitites in Eq. (14); w is nothing other than the component, along the z-body axis, of the inertial velocity of the center of the panel (slice). The authors arrive at their incorrect equation (45) by a convoluted and erroneous argument following Eq. (40). The source of the error appears to be an invalid statement in Eq. (41). The vector $\boldsymbol{r}$ cannot be both as in Eq. (14) and in Eq. (41); according to the definition of $\boldsymbol{r}$ preceding Eq. (14), $\boldsymbol{r}$ is constant in the body-fixed frame, so that $\dot{\boldsymbol{r}} = 0$, whereas in Eq. (41) $\dot{\boldsymbol{r}} \neq 0$. The incorrect value of $\dot{w}$ is then substituted into Eq. (38), which is the strip-theory application of an exact theoretical result. An erroneous expression for the $\mathrm{d}Z_I$ force component is then obtained in Eq. (46). It should read

$$\mathrm{d}Z_I = -\rho[k_3'(\dot{W} + \dot{P}y - \dot{Q}x) - k_1'Qu + k_2'Pv]A\,\mathrm{d}\xi \tag{3}$$

This error carries through into Eq. (47) and, of course, in a similar way, into expressions for the X_I and Y_I forces. One consequence is that the aerodynamic derivatives Z_q, M_q, Y_r, and N_r will all have extra erroneous terms related to terms such as Qu in Eq. (1).

When applied to the steady rotating arm experiment, to which the paper refers, the force Z_I for an axisymmetric body would correctly read [noting that only the Q and u components in Eq. (3) are different from zero in that case]:

$$\mathrm{d}Z_I = [\rho k_1'Qu]A\,\mathrm{d}\xi \tag{4}$$

If the authors have found that additional terms in this equation are needed to produce agreement with experiment then that is an empirical consequence of the real fluid properties, and cannot be justified by recourse to an ideal-fluid theory.

We have not calculated any examples to see what effect the errors in the paper have on numerical results.

References

[1]Jones, S. P. and DeLaurier, J. D., "Aerodynamic Estimation Techniques for Aerostats and Airships," *Journal of Aircraft*, Vol. 20, Feb. 1983, pp. 120-126.

Submitted Dec. 7, 1984.

*University Professor Emeritus.
†Assistant Professor.

Reply by Authors to B. Etkin and G. D'Eleuterio

S. P. Jones*
TCOM Corporation, Columbia, Maryland
and
J. D. DeLaurier†
University of Toronto, Toronto, Canada

CRITICISM by Professor Etkin is not taken lightly, especially when it is so unequivocal. In this case we maintain that our analysis is correct. The question raised is fundamental, of importance not only to the dynamics of airships, but of missiles, submarines, and wide-body aircraft. It is appropriate, therefore, that it should be aired in a public forum.

The terms to which objection is taken represent the convective acceleration, $-(\boldsymbol{V}\cdot\nabla)\boldsymbol{V} = -\omega\times\boldsymbol{V}$ in our coordinate system. Omitting these terms assumes apparent mass behaves as a rigid body for which $\dot{r} = 0$. While this appears to be the case in the solutions given by Lamb[1] for a rotating and translating body, it is true only at closure, to which his solutions apply. Lamb's solutions cannot be obtained from strip theory without including the convective acceleration, which vanishes at closure. For nonclosure, to which our analysis applies, these terms must be included. Confirmation of this is found in Brown,[2] Schlichting and Truckenbrodt,[3] and Multhopp,[4] whose formulations agree with ours, including the term due to convective acceleration.

To illustrate, consider a body of revolution rotating and translating in its vertical plane. For simplicity, assume a slender body for which

$$K = k_3 = 1; \qquad k_1 = 0$$

and a reference point at the nose about which the body rotates. In the nomenclature of our paper,

$$w(\xi) = W + \xi Q, \qquad u = U \tag{a}$$

Neglecting cross-flow friction drag, the vertical force is

$$Z = Z_a + Z_I \tag{b}$$

Submitted Feb. 8, 1985.

*Advisory Scientist. Member AIAA.
†Associate Professor, Institute for Aerospace Studies, Associate Fellow AIAA.

The first right-hand term is obtained from our Eq. (24) which can be written for these conditions and small angles of attack as

$$Z_a = -\rho U \int_0^{\ell_h} w(\xi) \frac{dA}{d\xi} d\xi = -\rho U [I_1 W + I_3 Q] \qquad (c)$$

The inertial term is derived from our Eq. (38),

$$dZ_I = -\rho [k_3 \dot{w} - k_1 UQ] A d\xi$$

which, for a slender body, becomes

$$dZ_I = -\rho \dot{w} A d\xi \qquad (d)$$

The present controversy concerns the definition of $\dot{w}$ derived from Eq. (a). We consider it to be

$$\dot{w} = \dot{W} + \xi \dot{Q} + \dot{\xi} Q$$

$$= \dot{W} + \xi \dot{Q} + \underline{UQ} \qquad (e)$$

Etkin and D'Eleuterio object to the last term, which is underlined for identification. Using Eq. (e) in Eq. (d) yields

$$Z_I = -\rho [I_2 \dot{W} + I_4 \dot{Q} + \underline{I_2 UQ}] \qquad (f)$$

and, from Eq. (b),

$$Z = -\rho [UWI_1 + UQI_3 + \dot{W} I_2 + \dot{Q} I_4 + \underline{UQI_2}] \qquad (g)$$

The I_i terms, defined in our Eqs. (35) and (36), are hull integrals, which are functions of ℓ_h. At closure

$$I_1 = 0$$

$$I_2 = \mathrm{V} = \text{volume}$$

$$I_3 = -\mathrm{V}$$

For an ellipsoid of revolution with the volumetric center at ℓ_{cv}

$$I_4 = \ell_{cv} \mathrm{V}$$

Consider now the rotating-arm experiment in completely attached flow.

$$\dot{W} = \dot{Q} = 0$$

$$Z = -\rho [UWI_1 + UQI_3 + \underline{UQI_2}] \qquad (h)$$

$$= -\rho [-UQ\mathrm{V} + \underline{UQ\mathrm{V}}] = 0 \qquad (i)$$

which agrees with the classical theory. Etkin and D'Eleuterio would omit the underlined term and their result would be

$$Z = \rho UQ\mathrm{V} \quad \text{incorrect} \qquad (j)$$

Equation (h) is substantiated by Schlichting and Truckenbrodt's Eq. (5-28) of Ref. 3, derived by integration of the pressure computed from the potential field. The same relationship was derived by Multhopp[4] from momentum considerations. Upon using our notation, that analysis gives

$$\frac{dZ}{d\xi} = -\rho U^2 \frac{d}{d\xi}\Big[\alpha(\xi) A(\xi)\Big] \qquad (k)$$

$$= -\rho U \frac{d}{d\xi}\Big[w(\xi) A(\xi)\Big]$$

$$= -\rho U \Big[(W + \xi Q) \frac{dA}{d\xi} + QA\Big]$$

$$Z = -\rho [UWI_1 + UQI_3 + UQI_2] \qquad (l)$$

This agrees with Eq. (h), including the offending term.

Similar agreement is found with Brown,[2] who used the unsteady Bernoulli equation to include the effect of $\dot{W}$ and $\dot{Q}$. To make the comparison we note that

$$I_1 = A_h, \qquad I_2 + I_3 = A_h \ell_h, \qquad W = \alpha U$$

where A_h is the base area, being the cross-sectional area at the integration limit, ℓ_h. Substitution in Eq. (g) gives Brown's results in our coordinate system and notation.

$$Z = -\rho [U^2 \alpha A_h + UQA_h \ell_h + \dot{\alpha} U \mathrm{V} + \dot{Q} \ell_{cv} \mathrm{V}] \qquad (m)$$

Brown's value of Z_q for a slender body without fins agrees with ours, being

$$Z_q = -\rho U (I_3 + I_2)$$

$$= -\rho U A_h \ell_h \qquad (n)$$

It is in this parameter that the supposed error would appear. According to Etkin and D'Eleuterio, the error is

$$\Delta Z_q = -\rho U I_2 \qquad (o)$$

In nondimensional terms for a slender body at closure it is

$$\Delta \hat{Z}_q = -4 \hat{I}_2 = -4\mathrm{V}/(S\bar{c}) \qquad (p)$$

For a less slender body, such as an aerostat, our Eq. (63) would be modified by subtraction of the differential

$$\Delta \hat{Z}_q = -4 k_3' \hat{I}_2 \cos\alpha \qquad (q)$$

This differential for a Family II aerostat at zero incidence is about -1.5—a significant value which should be detectable by experiment. The agreement of our Eq. (63) with the rotating-arm experiments, as shown in Fig. 5, cannot be dismissed as irrelevant.

We would have welcomed our critics' presenting analyses or citing literature supporting their contention. We believe our formulations are correct within the limits of slender-body theory and consistent with the analyses and results of researchers cited in our references.

References

[1]Lamb, H., "On the Motion of Solids Through a Liquid: Dynamics Theory," *Hydrodynamics,* 6th Ed., Dover Publications, New York, 1945, pp. 166-170.

[2]Brown, C. E., "Aerodynamics of Bodies at High Speeds," *Aerodynamic Components of Aircraft at High Speeds,* edited by A. F. Donovan and H. R. Lawrence, Princeton University Press, Princeton, NJ, 1957, pp. 277-280.

[3]Schlichting, H. and Truckenbrodt, E., "Aerodynamics of the Fuselage," *Aerodynamics of the Airplane,* McGraw-Hill Book Co., New York, 1979, p. 344.

[4]Multhopp, H., "Aerodynamics of the Fuselage," NACA TM 1036, 1942.